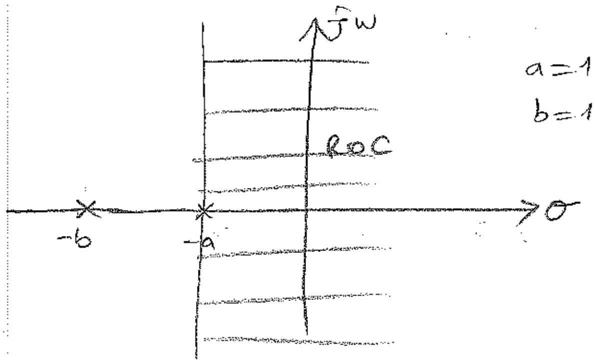
Example

If $H(s)$ is a stable and casual system transfer function is

$$H(s) = \frac{1}{(s+a)(s+b)}$$ determine arbitrary values for a&b and ROC.

poles are $-a, -b$



$a = 1$
$b = 16$

$Re\{s\} > -1$

The z-Transform

z-Transform is used for discreate signals.

$$\boxed{X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}}$$

$X(z) \longrightarrow$ z-transform of $x[n]$

$x[n] \xleftrightarrow{z} x[z]$

$z = re^{j\Omega} \longrightarrow$ complex number

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \longrightarrow x[z] = \sum_{n=-\infty}^{\infty} x[n] (re^{j\Omega})^{-n}$$

$$= X(z) = \sum_{-\infty}^{\infty} (x[n] r^{-n}) e^{-j\Omega n}$$

$$\boxed{X(z) = FT\{x[n] r^{-n}\}}$$

Consider z transform of $x[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

This summation converges for a set of z values the region in complex plane where $X(z)$ have finite summation is called ROC for $X(z)$.

Example

$x[n] = \alpha^n u[n] \quad X(z) = ? \quad ROC = ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-\infty}^{\infty} \alpha^n u[n] z^{-n} = \sum_{n=0}^{\infty} \alpha^n z^{-n}$$

Note

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \ |r| < 1$$

$$\sum_{n=0}^{\infty} (\underbrace{\alpha z^{-1}}_{r})^n = \frac{1}{1-\alpha z^{-1}}$$

$|\alpha z^{-1}| < 1 \qquad |z| > \alpha$