$$\frac{d^2\delta(t)}{dt^2} \underset{\text{derivative}}{\overset{\text{integral}}{\rightleftarrows}} \frac{d\delta(t)}{dt} \underset{\text{derivative}}{\overset{\text{integral}}{\rightleftarrows}} \delta(t) \underset{\text{derivative}}{\overset{\text{integral}}{\rightleftarrows}} u(t) \underset{\text{derivative}}{\overset{\text{integral}}{\rightleftarrows}} \int_{-\infty}^{t} u(t)\,dt$$

To find the homogenous part of the solution, equate the input ($x(t)$ to zero)

The particular part of the solution is obtained for a given specific $x(t)$ (input)

Example

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2x(t) + \frac{dx(t)}{dt}$$

Solution

Homogenous part

$$\frac{d^2y}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 0$$

$$\boxed{y_h(t) = c_1 e^{-3t} + c_2 e^{-2t}}$$

Characteristic equation $\Rightarrow r^2+5r+6=0$

$(r+2)(r+3)=0$

$r_1=-2,\ r_2=-3$

Particular solution

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2x(t) + \frac{dx(t)}{dt}$$

$x(t)$	$y_p(t)$
K	c
t	$c_1 t + c_2$
e^{-at}	$c e^{-at}$
t^n	$c_1 t^n + c_2 t^{n-1} + \dots + c_n$
$\cos(wt+\theta)$	$c_1\cos(wt) + c_2\sin(wt)$

$\boxed{\text{for } x(t) = e^{-t}}$

$y_h(t)$ was found as $y_h(t) = c_1 e^{-3t} + c_2 e^{-2t}$

$\boxed{y_p(t) = Ke^{-t}}$ by substitution in differential equation.

$$Ke^{-t} - 5Ke^{-t} + 6Ke^{-t} = 2e^{-t} - e^{-t}$$

$$2Ke^{-t} = e^{-t} \Rightarrow \boxed{K = 1/2}$$

so, the particular solution part is;

$$\boxed{y_p = \frac{1}{2} e^{-t}}$$

Then the general solution is obtained as

$$\boxed{y(t) = y_h(t) + y_p(t) = c_1 e^{-3t} + c_2 e^{-2t} + \frac{1}{2} e^{-t}}$$

The coefficients in general solution can also be obtained if the initial conditions are given such as $y(0)=1$, $y(-1)=0$