Thus for a periodic sequence Fourier Transform of $x[n]$

$$\boxed{X[\Omega] = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}}$$

Inverse Fourier Transform for $X(\Omega)$

$$\boxed{x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega}$$

for the convergence of $X(\Omega)$;

if $X(\Omega)$ doesn't go to ∞ then $|X(\Omega)| < \infty$

$$X(\Omega) < \infty \longrightarrow \left| \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \right| < \infty$$

$$\boxed{\sum_{n=-\infty}^{\infty} |x[n]| < \infty \quad x[n] \text{ will converge and } X(\Omega) \text{ exists.}}$$

Example

$x[n] = \delta(n-n_0) \qquad X(\Omega) = ?$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} \delta(n-n_0) e^{-j\Omega n} = \boxed{e^{-j\Omega n_0}}$$

Thus $\delta(n-n_0) \xleftrightarrow{F.T.} e^{-j\Omega n_0}$

$\delta(n+n_0) \xleftrightarrow{F.T.} e^{j\Omega n_0} \qquad n>0$

Using I.F.T

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega$$

$$\delta(n-n_0) = \frac{1}{2\pi} \int_{2\pi} e^{-j\Omega n_0} e^{j\Omega n} d\Omega$$

$$\boxed{2\pi\, \delta(n-n_0) = \int_{2\pi} e^{j\Omega(n-n_0)} d\Omega}$$

Widely used equation

The Laplace Transform

Laplace Transform is defined for continuous time signals and it can be thought as a generalisation of continuous time Fourier Transform.

Laplace Transform of $x(t)$ is defined as;

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt} \qquad s \longrightarrow \text{complex variable } (s = \sigma + j\omega)$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-(\sigma+j\omega)t} dt = \int_{-\infty}^{\infty} \left(x(t) e^{-\sigma t}\right) e^{-j\omega t} dt = FT\left(x(t) e^{-\sigma t}\right)$$