Note

$$\underbrace{e^{jw_0 t}}_{x(t)} \longleftrightarrow \underbrace{2\pi\delta(w-w_0)}_{X(w)}$$

$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$2\pi\delta(w-w_0) = \int_{-\infty}^{\infty} e^{jw_0 t}\, e^{-jwt}\, dt$$

$$2\pi\delta(w-w_0) = \int_{-\infty}^{\infty} e^{-j(w-w_0)t}\, dt$$

$$\boxed{\delta(w-w_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j(w-w_0)t}\, dt}$$

Properties of Fourier Transform

$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$X(w)$ → a complex function

$x(t)\, e^{-jwt}$ → continuous

$X(w) = a(w) + jb(w)$

where $|X(w)|$ (Absolute value of $X(w)$)

and $\angle X(w)$ (Phase value of $X(w)$)

$$\boxed{|X(w)| = \sqrt{a^2(w) + b^2(w)}}$$

$$\boxed{\angle X(w) = \tan^{-1}\frac{b(w)}{a(w)}}$$

Ex.

$$y(t) = \frac{dx(t)}{dt}$$

$x(t) \xrightarrow{F.T} X(w)$

$y(t) \xrightarrow{F.T}$? in terms of $X(w)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)\, e^{jwt}\, dw$$

$$y(t) = \frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)\,\frac{d}{dt}\left(e^{jwt}\right) dw = \frac{1}{2\pi}\int_{-\infty}^{\infty} (jw)\, X(w)\, e^{jwt}\, dw$$

$$y(t) = \frac{jw}{2\pi}\int_{-\infty}^{\infty} X(w)\, e^{jwt}\, dw$$

in general form

$$\boxed{\frac{d^n x(t)}{dt^n} \xleftrightarrow{FT} (jw)^n X(w)}$$

Summary

$x(t) \longrightarrow$ periodic signal $x(t) = x(t+T)$

Fourier Series Representation

Synthesis equation $x(t) = \sum_{k=-\infty}^{\infty} x[k]\, e^{jk\frac{2\pi}{T}t}$

$k \rightarrow$ integer

$T \rightarrow$ real number (Period)

$$x[k] = \frac{1}{T}\int_{T} x(t)\, e^{-jk\frac{2\pi}{T}t}$$