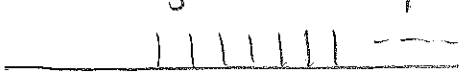
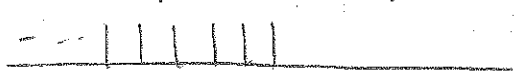
4) If $x[n]$ is a right sided sequence as shown below ROC is $|z| > r_0$



If $x[n]$ is a left sided sequence as shown below ROC is $|z| < r_0$



5) If $x[n]$ is a two sided signal, ROC is a ring in the z-plane.

Properties of the Z-Transform

$x_1[n] \xleftrightarrow{z} X_1[z]$ $\quad ROC_1$

$x_2[n] \xleftrightarrow{z} X_2[z]$ $\quad ROC_2$

1) Linearity

$$\alpha x_1[n] + \beta x_2[n] \xleftrightarrow{z} \alpha X_1[z] + \beta X_2[z] \quad ROC = ROC_1 \cap ROC_2$$

2) Time shifting

$$x[n] \xleftrightarrow{z} X[z] \quad ROC = R_x$$

$$x[n-n_0] \xleftrightarrow{z} z^{-n} X(z) \quad ROC = R_x$$

3) Frequency shifting

$$e^{j\omega_0 n} x[n] \xleftrightarrow{z} X(e^{-j\omega_0} z) \quad ROC = R_x$$

$$z_0^n x[n] \xleftrightarrow{z} X\left(\frac{z}{z_0}\right) \quad ROC = z_0 R_x$$

4) Time Reversal

$$x[-n] \xleftrightarrow{z} X\left(\frac{1}{z}\right) \quad ROC = \frac{1}{R_x}$$

5) Convolution Property

$$x_1[n] * x_2[n] \xleftrightarrow{z} X_1(z)\, X_2(z)$$

$$ROC = ROC_1 \cap ROC_2$$

6) Differentiation in the z-domain

$$n x[n] \xleftrightarrow{z} -z \frac{dX(z)}{dz} \quad ROC = R_x$$

7) The initial value theorem

If $x[n] = 0$, $n < 0$ then $\quad x[0] = \lim_{z \to \infty} X(z)$