Summary

$x(t) \longrightarrow$ Aperiodic

Fourier transform of $x(t)$

$$\boxed{X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt}}$$

Inverse Fourier transform for $x(t)$

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w) e^{jwt} dw}$$

Note: if $X(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$

then $x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X(w) e^{jwt} dw$

$c_1 c_2 = \frac{1}{2\pi} \Rightarrow$ if $c_1 = 1$ then $c_2 = \frac{1}{2\pi}$

if $c_1 = \frac{1}{\sqrt{2\pi}}$, $c_2 = \frac{1}{\sqrt{2\pi}}$

Convergence of Fourier Transform

Every signal may not have fourier Transform (F.T.)

in general $\int_{-\infty}^{\infty} x(t) e^{-jwt} dt$

$$\underbrace{\left| \int_{-\infty}^{\infty} x(t) e^{-jwt} dt \right|}_{\text{finite value}} < \infty \qquad \left| \int_{-\infty}^{\infty} x(t) e^{-jwt} dt \right| \leq \int_{-\infty}^{\infty} |x(t)| \left| e^{-jwt} \right| dt < \infty$$

Note:

$|e^{j\theta}| = 1$

Since $e^{j\theta} = \cos\theta + j\sin\theta$

$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = \boxed{1}$

Hence $\int_{-\infty}^{\infty} |x(t)| \underbrace{|e^{-jwt}|}_{1} dt < \infty$

$$\boxed{\int_{-\infty}^{\infty} |x(t)| dt < \infty} \longrightarrow$$ condition for convergence of F.T.

if a signal is not absolutely integrable then F.T doesn't exist.