Given a[k], b[k], A[0] are all real numbers. These can also be combined together and we can obtain complex Fourier series representation of periodic signals as

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} x[k]\, e^{jk\frac{2\pi}{T}t} \qquad x[k] = \frac{1}{T}\int_T x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt}$$

where x[k] is a complex number and real coefficients can be obtained using x[k].

$$A[k] = \left(x[k] + x^*[k]\right) \times 2 \qquad k \neq 0$$

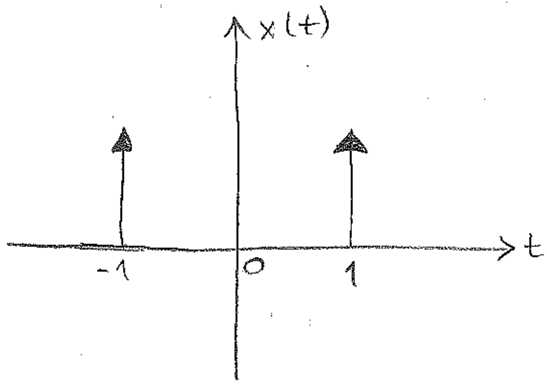
$$B[k] = \left(x[k] - x^*[k]\right) \cdot j$$

$$A[0] = x[0] \quad \text{OR} \quad x[k] = \frac{1}{2}\left(A[k] - j B[k]\right)$$

$$A(0) = x(0)$$

Example

One period of a periodic signal is given below



x(t) has period of 8 (given)

Find Fourier series representation

we can either use

$$x(t) = A[0] + \sum_{k=1}^{\infty} A[k] \cos\left(\frac{k2\pi}{T}t\right) + \sum_{k=1}^{\infty} B[k] \sin\left(\frac{k2\pi}{T}t\right)$$

or

we can use

$$x(t) = \sum_{k=-\infty}^{\infty} x[k]\, e^{jk\frac{2\pi}{T}t}$$

for the use of first; we evaluate the coefficients at first: T = 8 (given)

$$A[0] = \frac{1}{8}\int_T x(t)\, dt = \frac{1}{8}\int_{-4}^{4} \left(\delta(t+1) + \delta(t-1)\right) dt = \frac{1}{8}(1+1) = \boxed{\frac{1}{4}}$$

$$A[k] = \frac{2}{8}\int_{-4}^{4} \left(\delta(t+1) + \delta(t-1)\right) \cos\left(k\frac{2\pi}{T}t\right) dt$$

$$A[k] = \frac{1}{4}\left[\cos\left(\frac{k2\pi}{T}\right) + \cos\left(\frac{k2\pi}{T}\right)\right] = \boxed{\frac{1}{2}\cos\left(\frac{k2\pi}{T}\right)}$$

$$B[k] = \frac{1}{4}\int_{-4}^{4} \left(\delta(t+1) + \delta(t-1)\right) \sin\left(k\frac{2\pi}{T}t\right) dt$$

$$= \frac{1}{4}\left[\sin\left(\frac{k2\pi}{T}(-1)\right) + \sin\left(\frac{k2\pi}{T}(1)\right)\right]$$

$$= \boxed{\varnothing}$$