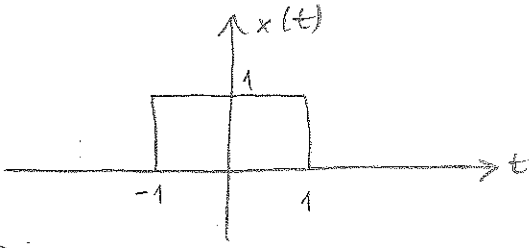
Since $x(t)$ is a single period of $\tilde{x}(t)$

It can be drawn as



$$X(\omega) = \int_{-1}^{1} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} 1 \cdot e^{-j\omega t}\, dt$$

$$= \frac{1}{-j\omega}\left(e^{-j\omega} - e^{j\omega}\right) = \boxed{\frac{2}{\omega}\sin(\omega)}$$

If the fourier series coefficients are desired to be evaluated

$$(\tilde{x}(t) \longrightarrow \tilde{x}(k))$$

$$\tilde{x}[k] = \frac{1}{T}\, X(\omega)\Big|_{\omega = k\omega_0} \qquad \omega_0 = \frac{2\pi}{T} \text{ where } T \text{ is given as } 4$$

$$\tilde{x}[k] = \frac{1}{4}\cdot\frac{2}{\omega}\sin\omega\Big|_{\omega = k\frac{2\pi}{4}}$$

$$\tilde{x}[k] = \frac{1}{\cancel{4}}\cdot\frac{\cancel{2}}{\frac{k\pi}{\cancel{2}}}\sin\left(\frac{k\pi}{2}\right)$$

$$\boxed{\tilde{x}[k] = \frac{1}{k\pi}\sin\left(\frac{k\pi}{2}\right)}$$

$$\tilde{x}(t) = \sum_k \tilde{x}[k]\, e^{jk\frac{2\pi}{4}t}$$

$$\tilde{x}(t) = \sum_k \frac{1}{k\pi}\sin\frac{k\pi}{2}\, e^{jk\frac{\pi}{2}t}$$

Evaluation of $\tilde{x}(t)$ from its Fourier Series coefficients...

Example

$$X(\omega) = 2\pi\,\delta(\omega - \omega_0)$$

$x(t) = ?$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega = \frac{1}{\cancel{2\pi}}\int_{-\infty}^{\infty} \cancel{2\pi}\,\delta(\omega - \omega_0)\, e^{j\omega t}\, d\omega$$

$$= \boxed{e^{j\omega_0 t}}$$

Note that

$$\boxed{e^{jk\omega_0 t} = 2\pi\,\delta(\omega - k\omega_0)}$$

Fourier Transform of aperiodic signal

$x(t) \longrightarrow$ periodic with period T

$$x(t) = \sum_k x[k]\, e^{jk\omega_0 t} \xrightarrow{FT} X(\omega) = \sum_k x[k]\, 2\pi\,\delta(\omega - k\omega_0)$$

Thus $\boxed{X(\omega) = 2\pi \sum_k x[k]\,\delta(\omega - k\omega_0)}$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{2\pi \sum_k x[k]\,\delta(\omega - k\omega_0)}_{X(\omega)}\, e^{j\omega t}\, d\omega = \sum_k x[k]\int_{-\infty}^{\infty}\delta(\omega - k\omega_0)\, e^{j\omega t}\, d\omega$$

$$x(t) = \sum_k x[k]\, e^{jk\omega_0 t}$$