Signal Energy and Power

Energy

Sum of squares of abs value of signal between a given interval.

$$E_c = \int_{t_1}^{t_2} |x(t)|^2 dt \Rightarrow \boxed{P_c = \frac{E_c}{t_2 - t_1}}$$

$$E_d = \sum_{n=-N}^{N} |x[n]|^2 \Rightarrow \boxed{P_d = \frac{E_d}{2N+1}}$$

Example

Evaluate the E_∞ and P_∞ for

a) $x_1(t) = \cos(t)$ b) $x_2[n] = e^{j\left(\frac{\pi}{2n} + \frac{\pi}{8}\right)}$

Solution

a) $$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \lim_{T\to\infty} \int_{-T}^{T} |\cos(t)|^2 dt = \lim_{T\to\infty} \int_{-T}^{T} \frac{1+\cos 2T}{2} dt$$

$$= \lim_{T\to\infty} \frac{I}{2}\Big|_{-T}^{T} + \frac{\sin 2T}{4}\Big|_{-T}^{T} = \frac{I}{2} + \frac{I}{2} + \frac{\sin 2T}{4} + \frac{\sin 2T}{4}$$

$$= T + \frac{\sin 2T}{2} = \boxed{\infty}$$

$$P_\infty = \frac{E_\infty}{2T} = \frac{T + \frac{\sin(2T)}{2}}{2T} = \frac{1}{2} + \frac{\sin 2T}{4T} = \boxed{\frac{1}{2}}$$

b) $$E_\infty = \sum_{n=-N}^{N} \left|e^{j(\pi/2n + \pi/8)}\right|^2$$

Note: $|x[n]| = \sqrt{x[n] \cdot x^*[n]}$ (x^*: conjugate)

$$E_\infty = \sum_{n=-N}^{N} \sqrt{e^{j(\pi/2n+\pi/8)} e^{-j(\pi/2n+\pi/8)}}^2$$

$$= \sum_{n=-N}^{N} \sqrt{e^0}^2 = \sum_{n=-N}^{N} 1 = \boxed{2N+1}$$

$$P_\infty = \frac{E_\infty}{2N+1} = \boxed{1}$$