The Inverse Laplace Transform

$x(t) \xleftrightarrow{\mathcal{L}} X(s)$

$x(t) = \mathcal{L}^{-1}\{X(s)\}$

$X(s) = \mathcal{L}\{x(t)\}$

$s = \sigma + j\omega$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

Inverse Laplace Transform Formula

$$x(t) = \frac{1}{j2\pi} \lim_{\omega \to \infty} \int_{\sigma - j\omega}^{\sigma + j\omega} X(s)e^{st}\,ds$$

But, since this formula is not useful, inverse laplace transform will be evaluated by using the L.T of some well known $\mathcal{L}$. Transforms.

Example

$X(s) = \dfrac{1}{(s+1)(s+2)} \quad Re\{s\} > -1 \quad x(t) = ?$

$\dfrac{1}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2}$ Multiply both sides by $(s+1)(s+2)$

$1 = A(s+2) + B(s+1)$

$-A + B = 0$
$2A + B = 1$

$\boxed{A = 1}$ $\boxed{B = -1}$

$X(s) = \dfrac{1}{s+1} - \dfrac{1}{s+2}$ ROC is $Re\{s\} > -1$

$e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \dfrac{1}{s+1}$, $Re\{s\} > -1$

$e^{-2t}u(t) \xleftrightarrow{\mathcal{L}} \dfrac{1}{s+2}$ $Re\{s\} > -2$

$$\boxed{\left(e^{-t} - e^{-2t}\right)u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+1)(s+2)}}$$