Example

$x(t) = e^{-\alpha|t|}$ $\quad X(s) = ?$

$\dot{x}(t) = e^{-\alpha|t|} \longrightarrow x(t) = \begin{cases} e^{-\alpha t} & t>0 \\ e^{-\alpha(-t)} & t<0 \end{cases}$

$\longrightarrow x(t) = \begin{cases} e^{-\alpha t} & t>0 \\ e^{\alpha t} & t<0 \end{cases} \longrightarrow x(t) = e^{-\alpha t}u(t) + e^{\alpha t}u(-t)$

$e^{-\alpha t}u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+\alpha}$ $\quad \underline{Re(s) > -\alpha}$

$e^{\alpha(t)}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} \int_{-\infty}^{\infty} e^{\alpha t}u(-t)e^{-st}dt = \int_{-\infty}^{\infty} e^{\alpha t}u(-t)e^{-st}dt$

$= \int_{-\infty}^{0} e^{\alpha t}e^{-st}dt = \int_{-\infty}^{0} e^{t(\alpha-s)}dt = \boxed{-\frac{1}{s-\alpha}}$

$Re\{\alpha - s\} > 0 \Rightarrow Re\{s\} < \alpha$

Thus $e^{\alpha t}u(-t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{1}{s-\alpha}$ $\quad \underline{Re\{s\} < \alpha}$

Hence; $e^{-\alpha|t|} \overset{\mathcal{L}}{\longleftrightarrow} \mathcal{L}\{e^{-\alpha t}u(t)\} + \mathcal{L}\{e^{\alpha t}u(-t)\}$

$\boxed{e^{-\alpha|t|} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+\alpha} - \frac{1}{s-\alpha}}$ $\quad \boxed{ROC \longrightarrow -\alpha < R(s) < \alpha}$

Properties of Laplace Transform

$x_1(t) \overset{\mathcal{L}}{\longleftrightarrow} X_1(s)$ $\quad ROC = R_1$

$x_2(t) \overset{\mathcal{L}}{\longleftrightarrow} X_2(s)$ $\quad ROC = R_2$

$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s)$ $\quad ROC = R$

1) Linearity $ax_1(t) + bx_2(t) \overset{\mathcal{L}}{\longleftrightarrow} aX_1(s) + bX_2(s)$

ROC $R_1 \cap R_2$

2) Time shifting $x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s)$ $\quad ROC = R$

$x(t-t_0) \overset{\mathcal{L}}{\longleftrightarrow} e^{-st_0}X(s)$ $\quad$ with $\quad ROC = R$

3) Shifting in the s-Domain

$x(t) \overset{\mathcal{L}}{\longleftrightarrow} x(s)$, $ROC = R$

$e^{s_0 t}x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s-s_0)$ $\quad ROC = R + Re\{s_0\}$

4) Time Scaling

$x(at) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{|a|}X\left(\frac{s}{a}\right)$, $\quad Roc = \frac{R}{a}$