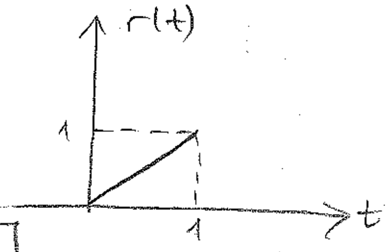
RAMP FUNCTION

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & \text{other} \end{cases}$$



$$\boxed{u(t) = \frac{dr(t)}{dt}, \quad r(t) = \int_{-\infty}^{t} u(t)\,dt}$$

$$\boxed{\delta(t) = \frac{du(t)}{dt}, \quad u(t) = \int_{-\infty}^{\infty} \delta(t)\,dt}$$

$$\boxed{r(t) \underset{\text{integration}}{\xrightarrow{\text{derivative}}} u(t) \underset{\text{integration}}{\xrightarrow{\text{derivative}}} \delta(t)}$$

$$\boxed{\delta(t) = \frac{\partial^2 r(t)}{dt^2}}$$

Example

If $f(t) = 2t^2+1$. Evaluate the results of the given operations.

a) $f(t)\,\delta(t-1) = ?$

$f(1)\cdot 1 = 2\cdot 1+1 = \boxed{3}$

b) $\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = ?$

$\Downarrow$

$f(0) = 2\cdot 0+1 = \boxed{1}$

c) $\int_{-\infty}^{\infty} f(t)\,\delta(t-2)\,dt = f(2) = 2\cdot 4+1 = \boxed{9}$

Example

Evaluate the results of given expressions

a) $\frac{d\,u(t-2)}{dt} = \boxed{\delta(t-2)}$

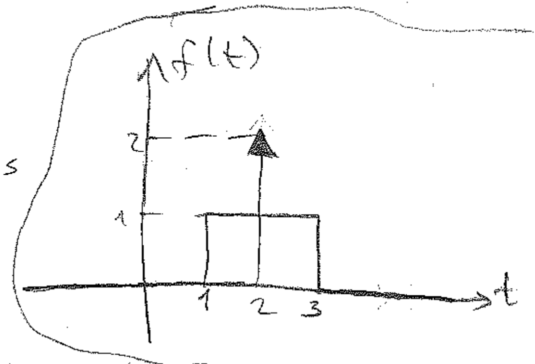
b) $\frac{d\,u(t^2+1)}{dt} = \boxed{\delta(t^2+1)(2t)}$

c) $\frac{d\,r(t^2+1)}{dt} = \boxed{u(t^2+1)(2t)}$

Example

Draw the graphs of given functions

a) $f(t) = u(t-1) - u(t-3) + \delta(t-2)$



Solution a)