$$z(t) = \sum_k \underbrace{x[k]\, y[k].T.}_{z[k]}\, e^{jk\frac{2\pi}{T}t} \quad , \quad z[k] = T\, x[k]\, y[k] \quad (\text{if } k=m)$$

$$x(t) * y(t) \xleftrightarrow{FSC} T\, x[k]\, y[k]$$

Representation of aperiodic Signals

The Continuous Time Fourier Transform

$\tilde{x}(t) \longrightarrow$ periodic signal

$x(t) \longrightarrow$ one period of the periodic $\tilde{x}(t)$ signal

$$\boxed{\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t-kT)}$$

$$\tilde{x}(t) \xleftrightarrow{FSC} \tilde{x}[k]$$

$$\tilde{x}[k] = \frac{1}{T}\int_T \tilde{x}(t)\, e^{-jk\frac{2\pi}{T}t}\, dt = \frac{1}{T}\int_{-\infty}^{\infty} x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$

$\longrightarrow$ one period of $\tilde{x}(t)$

$$T.\,\tilde{x}[k] = \int_{-\infty}^{\infty} x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$

$$x(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jkw_0 t}\, dt \quad , \quad w = kw_0$$

$$\boxed{x(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt}$$ (Fourier Transform of aperiodic signal $x(t)$)

On the other hand;

$$x(w) = T\,\tilde{x}(k) \qquad w = kw_0$$

$$\tilde{x}[k] = \frac{x(kw_0)}{T}$$

$$\tilde{x}(t) = \sum_k \tilde{x}[k]\, e^{j\frac{2\pi}{T}t} = \sum_k \frac{x(kw_0)}{T}\, e^{jk\frac{2\pi}{T}t} \qquad T = \frac{2\pi}{w_0}$$

$$\tilde{x}(t) = \sum_k \frac{w_0}{2\pi}\, x(kw_0)\, e^{jkw_0 t}$$

for $T \to \infty$ $\quad \tilde{x}(t) = x(t) \quad w_0 = \frac{2\pi}{T}$

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(w)\, e^{jwt}\, dw}$$ (Inverse Fourier Transform of aperiodic signal)