Example

$$\phi[n] = e^{jk\frac{2\pi}{N}n} \quad k,n \in \mathbb{Z}$$

$$\phi[n] \stackrel{?}{=} \phi[n+N]$$

$$\phi[n+N] = e^{jk\frac{2\pi}{N}(n+N)} = e^{jk\frac{2\pi}{N}n} \cdot \underbrace{e^{jk\frac{2\pi}{\cancel{N}}\cancel{N}}}_{1} = e^{jk\frac{2\pi}{N}n}$$

Thus $\phi[n] = \phi[n+N]$

if $\underset{\substack{\downarrow \\ \text{integer value}}}{n = m.N}$ $\quad \phi[n] = e^{jk\frac{2\pi m}{\cancel{N}}\cancel{N}} = e^{jk2\pi m} = 1$

Example

Consider the summation

$$\sum_{n=0}^{N-1} r^n = 1 + r + r^2 + r^3 + \cdots = \frac{1-r^n}{1-r} \quad \text{if } r \neq 1$$

$$= 1.N \quad \text{if } r = 1$$

using the above information, calculate $\sum_{n=0}^{N-1} \left(e^{jk\frac{2\pi}{N}}\right)^n = ?$

$$\sum_{n=0}^{N-1} \left(e^{jk\frac{2\pi}{N}}\right)^n \longrightarrow \sum_{m=0} \left(e^{jm\cancel{N}\frac{2\pi}{\cancel{N}}}\right)^n \longrightarrow \sum_{m=0}^{N-1} \underbrace{\left(e^{jm2\pi}\right)^n}_{1} = \sum_{n=0}^{N-1} 1 = N$$

if $k = 0, \pm N, \pm 2N$

if $k \neq m.N$

$$\sum_{n=0}^{N-1} \left(e^{jk\frac{2\pi}{N}}\right)^n = \frac{1-\left(e^{jk\frac{2\pi}{N}}\right)^N}{1-e^{jk\frac{2\pi}{N}}} = \frac{1-e^{jk\frac{2\pi}{\cancel{N}}\cancel{N}}}{1-e^{jk\frac{2\pi}{N}}} = \frac{1-1}{1-e^{jk\frac{2\pi}{N}}} = 0$$

Hence;

$$\sum_{0}^{N-1} e^{jk\frac{2\pi}{N}n} = \begin{cases} N & \text{if } k = \pm mN \\ 0 & \text{else} \end{cases}$$

Example

$x[n]$ periodic signal is given below Find Fourier Series Representation