$$\sum_{k=-\infty}^{n} h[k] = \sum_{k=-\infty}^{n-1} h[k] + h[n]$$

$$s[n] = s[n-1] + h[n]$$

$$h[n] = s[n] - s[n-1]$$

for continuous time

$$h(t) = \frac{ds(t)}{dt} \qquad s(t) = \int_{-\infty}^{\infty} h(t)\,dt$$

Definition of Causal LTI Systems with Differantial Equations and Difference Equations

A continuous time system can be expressed as linear constant coefficient differantial equation.

$$\frac{dy(t)}{dt} + 2(y(t)) = x(t)$$

$$y(t) = \underbrace{y_h(t)}_{\text{homogenous part}} + \underbrace{y_p(t)}_{\text{particular part}}$$

Let's find the homogenous part first:

$$\frac{dy(t)}{dt} + 2y(t) = 0 \qquad y_h(t) = Ae^{st} \text{ is known}$$

Note: y(t) is evaluated by summation of it's homogenous and particular parts its called the general solution.

$$sAe^{st} + 2Ae^{st} = 0 \Rightarrow Ae^{st}(s+2) = 0$$

$$\boxed{s=-2} \qquad \boxed{y_h(t) = Ae^{-2t}}$$

Let's find the particular part

if $x(t) = Ke^{3t}u(t)$

$$y_p(t) = Ye^{3t} \quad t \geq 0$$

$$3Ye^{3t} + 2Ye^{3t} = Ke^{3t}$$

$$5Y = K \qquad \boxed{Y = K/5}$$

$$\boxed{y(t) = Ae^{-2t} + \frac{K}{5}e^{3t} \quad \text{for } t \geq 0}$$

For the causal system initial value is satisfied as zero for t=0. By using this property value of A can be evaluated.

for casual systems output it zero for $t \leq 0$ if $t=0$

$$y(0) = 0 \Rightarrow A + \frac{K}{5} = 0 \qquad \boxed{A = -\frac{K}{5}}$$

$$y(t) = \left[-\frac{K}{5}e^{-2t} + \frac{K}{5}e^{3t}\right]u(t)$$

$$\boxed{y(t) = -\frac{K}{5}e^{-2t} + \frac{x(t)}{5}}$$