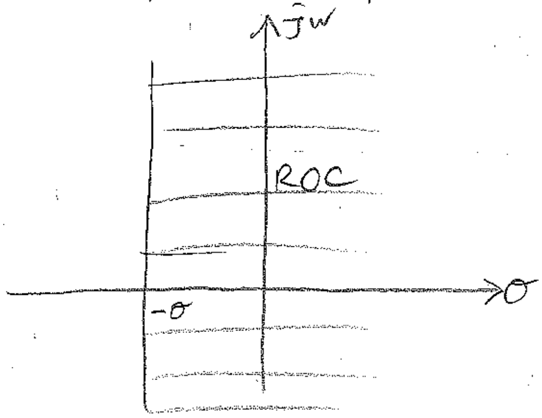
Laplace Transform of $x(t)$ is F.T. of $x(t)e^{-\sigma t}$ for convergence of $X(s)$;

$$\int_{-\infty}^{\infty} |x(t)e^{-\sigma t}| < \infty$$ can be satisfied for a range of σ values for which

we call this region as ROC (Region of convergence)

Since $s\,(\sigma + j\omega)$ can be defined on S plane ROC of $X(s)$ will also be defined on s-plane.



Example

$x(t) = e^{-\alpha t}u(t)$ $X(s) = ?$ $ROC = ?$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} e^{-\alpha t}u(t)e^{-st}\,dt$$

$$X(s) = \int_{0}^{\infty} e^{-\alpha t}e^{-st}\,dt = \int_{0}^{\infty} e^{-\alpha t}e^{-(\sigma + j\omega)t}\,dt$$

$$= \int_{0}^{\infty} e^{-t\underbrace{(\alpha+\sigma)}}e^{-j\omega t}\,dt$$

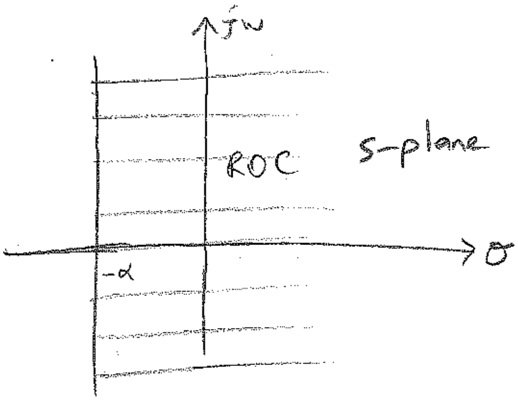
$\hookrightarrow$ $\alpha + \sigma > 0$ must be satisfied, otherwise it doesn't converge.

if $\alpha + \sigma > 0$ then $\sigma > -\alpha$

$Re\{s\} > -\alpha$

$$X(s) = \int_{-\infty}^{\infty} e^{-\alpha t}e^{-st}\,dt = \int_{-\infty}^{\infty} e^{-t(s+\alpha)}\,dt$$

$$= -\frac{1}{s+\alpha}e^{-t(s+\alpha)}\Big|_{0}^{\infty} = \boxed{\frac{1}{s+\alpha}}, \quad Re\{s\} > -\alpha$$



Example

$x(t) = -e^{-\alpha t}u(-t)$ $X(s) = ?$ $ROC = ?$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt = \int_{-\infty}^{\infty} -e^{-\alpha t}u(-t)e^{-st}\,dt = \int_{-\infty}^{0} -e^{-\alpha t}e^{-st}\,dt$$

$$= \int_{-\infty}^{0} -e^{-t(\alpha+s)}\,dt \Rightarrow X(s) = \int_{-\infty}^{0} -e^{-t(\alpha+s)}\,dt \quad Re\{s+\alpha\} < 0$$

$\hookrightarrow$ ROC condition

$$X(s) = \frac{1}{s+\alpha}e^{-t(s+\alpha)}\Big|_{-\infty}^{0}, \quad Re\{s\} < -\alpha$$

$$\boxed{X(s) = \frac{1}{s+\alpha}}$$

Thus $-e^{-\alpha t}u(-t) \xleftrightarrow{L.S} \frac{1}{s+\alpha}$ $Re\{s\} < -\alpha$

$e^{-\alpha t}u(t) \xleftrightarrow{L.S} \frac{1}{s+\alpha}$ $Re\{s\} > -\alpha$